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1991 J. Phys. A: Math. Gen. 24 4549

(http://iopscience.iop.org/0305-4470/24/19/018)

J. Phys. A: Math. Gen. 24 (1991) 4549–4555. Printed in the UK

Comments on the quantum statistical system of 'monopoles': an equivalence with the Liouville theory

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Received 4 March 1991, in final form 15 May 1991

Abstract. We consider a two-dimensional field theory whose associated statistical mechanical system is a gas of strings of electric poles ('monopoles'). We show that the corresponding partition function is that of the Liouville theory in the presence of a uniform neutralizing background. A general mapping between arbitrary correlation functions of the two theories is established. The connection with the two-dimensional one-component plasma is also discussed.

Besides the formal aspects, the two-dimensional Liouville field theory (LT) attracts considerable interest due to the important role played by it in the quantum string theory [1]. The difficulties in the quantization of the LT are associated with the monotonicity of the interaction Lagrangian and the consequent absence of a finite constant minimum of the classical potential.

Some quantization schemes for the LT have been proposed [2, 3]. A quantization prescription in terms of the corresponding statistical mechanical version was proposed in [3]. From the statistical mechanical point of view, the quantization problems of the LT are associated with the non-neutrality of the corresponding one-component gas, which implies that the quantum statistical system is (infrared) unstable. In order to prevent the infrared instability, the Liouville field is shifted by its mean value over a finite volume. This procedure implies the introduction of a uniform background and the corresponding statistical system is neutral (charge screening). The thermodynamical limit is performed at the end of all computations and the resulting quantum theory is translational invariant [3].

In this paper we will consider a two-dimensional field theory defined by the following Lagrangian density

$$\mathscr{L}(x)=\tfrac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x)-\alpha_0{:}\exp\left\{\beta\int_{x,C}^{\infty}\varepsilon_{\mu\nu}\partial^\nu\phi(z)\,\mathrm{d}z^\mu\right\}{:} \qquad (1)$$

where C is an arbitrary integration path and the notation :{...}: means normal ordering of the interaction Lagrangian. The above Lagrangian density is invariant under translations of the $\phi(x)$ field.

As we will see, in the two-dimensional Euclidean space the associated statistical system is a gas of strings of 'electric poles' ('monopoles') [4], with imaginary charges, interacting via the Coulomb potential. Considering the system confined in a finite volume, we show that the grand partition function of this system is identical to that obtained for the charge-screened LT in [3]. Thus, as a consequence of the symmetry under the interchange of charges and 'monopoles', a general one-to-one mapping between arbitrary correlation functions of the two theories is established.

To begin with, consider the Euclidean vacuum functional given by

$$\mathscr{Z}[0]_{\mathscr{M}} = \mathscr{Z}_0^{-1} \int \mathscr{D}\phi \exp\left\{ -\int d^2z [\tfrac{1}{2}(\partial_\mu \phi(z))^2 + \alpha_0 \mathscr{F}(z) e^{i\beta\Sigma(z)}] \right\} \tag{2}$$

where the Euclidean potential is complex, $\mathscr{Z}_0$ is the free ($\alpha_0 = 0$) vacuum functional, Σ is the line integral of a topological conserved current,

$$\Sigma(x) = \int_{x,c}^{\infty} \varepsilon_{\mu\nu} \partial^\mu \phi(z) \, dz^\mu \tag{3}$$

and the normal ordering of the interaction Lagrangian is defined by

$$\int :e^{\beta\Sigma(z)}: d^2z = \int \mathscr{F}(z) e^{\beta\Sigma(z)} d^2z. \tag{4}$$

In order to ensure the infrared stability (neutrality) of the corresponding statistical system, we will write the line integral (3) in a more general way as

$$\Sigma(z) = \lim_{\Omega\to\infty} \Sigma(z,\Omega) = \lim_{\Omega\to\infty} \int_{z,c}^{\Omega} \varepsilon_{\mu\nu} \partial^\nu \phi(\xi) \, d\xi^\mu \tag{5}$$

where Ω is an arbitrary point. As we will see, this implies the introduction of a non-uniform neutralizing background of 'monopoles'. The statistical mechanical description of the theory is obtained by making the following gas expansion [5]:

$$\exp\left(\alpha_0 \int d^2z f(z) e^{i\beta\Sigma(z,\Omega)} \right) = \sum_{\mathscr{N}=0}^{\infty} \frac{(-\alpha_0)^{\mathscr{N}}}{\mathscr{N}!} \int \prod_{j=1}^{\mathscr{N}} d^2z_j \mathscr{F}(z_j) \exp\left(i\beta \sum_{j=1}^{\mathscr{N}} \Sigma(z_j,\Omega_j) \right). \tag{6}$$

The vacuum functional is then given by

$$\mathscr{Z}[0]_{\mathscr{M}} = \sum_{\mathscr{N}=0}^{\infty} \frac{(-\alpha_0)^{\mathscr{N}}}{\mathscr{N}!} \int \prod_{j=1}^{\mathscr{N}} d^2z_j \, \Gamma(z_1,\Omega_1;\ldots;z_{\mathscr{N}},\Omega_{\mathscr{N}}) \tag{7}$$

where

$$\begin{aligned} \Gamma(z_1,\Omega_1;\ldots;z_{\mathscr{N}},\Omega_{\mathscr{N}}) &= \mathscr{Z}_0^{-1} \prod_{j=1}^{\mathscr{N}} \mathscr{F}(z_j) \int \mathscr{D}\phi \exp\left\{ \int d^2z \left[-\tfrac{1}{2}(\partial_\mu\phi(z))^2 \right.\right. \\ &\quad \left.\left. + i\beta \sum_{j=1}^{\mathscr{N}} \int_{z_j,C_j}^{\Omega_j} \varepsilon_{\mu\nu} \partial^\nu_{(\xi)} \delta(\xi - z) \, d\xi_\mu \, \phi(z) \right] \right\}. \end{aligned} \tag{8}$$

Performing the quadratic functional integration in (8), we obtain

$$\begin{aligned} \Gamma(z_1,\Omega_1;\ldots;z_{\mathscr{N}},\Omega_{\mathscr{N}}) &= \prod_{i=1}^{\mathscr{N}} \mathscr{F}(z_i) \exp\left\{ \tfrac{1}{2} \int d^2z \, d^2z' \, \rho_{\mathscr{M}}(z;z_1,\Omega_1;\ldots;z_{\mathscr{N}},\Omega_{\mathscr{N}}) \right. \\ &\quad \left. \times D(z-z') \rho_{\mathscr{M}}(z';z_1,\Omega_1;\ldots;z_{\mathscr{N}},\Omega_{\mathscr{N}}) \right\} \end{aligned} \tag{9}$$

where $D(z)$ is the infrared and ultraviolet regularized massless Green function [6]

$$D(z)=\lim_{\epsilon,\mu\to 0}\left\{-\frac{1}{4\pi}\ln \mu^2(|z|^2+|\epsilon|^2)\right\} \tag{10}$$

and the neutral charge density is given by

$$\rho_{\mathscr{U}}(z;z_1,\Omega_1;\ldots;z_{\mathcal{N}},\Omega_{\mathcal{N}})=\mathrm{i}\beta\sum_{j=1}^{\mathcal{N}}\partial^{\nu}_{(z)}\int_{z_j,C_j}^{\Omega_j}\varepsilon_{\mu\nu}\delta(\xi-z)\,\mathrm{d}\xi_\mu . \tag{11}$$

The charge distribution above corresponds to a density of $\mathcal{N}$ 'magnetic monopoles' pairs located at z_j and Ω_j ($j=1,2,\ldots,\mathcal{N}$), with a singular string connecting these two points. Observe the distinction with the case of point charge density [3], where the charge distribution is given only in terms of delta functions (local singularities). Supposing that the $\mathcal{N}$ curves C_i never intercept each other nor have common segments, we obtain the following expression for the vacuum functional:

$$\begin{aligned}\mathscr{Z}[0]_{\mathscr{U}}=\sum_{\mathcal{N}=0}^{\infty}\frac{(-\alpha_0)^{\mathcal{N}}}{\mathcal{N}!}\int\prod_{i=1}^{\mathcal{N}}\mathrm{d}^2z_i\,\mathscr{F}(z_i)\exp\bigg\{&-\frac{\beta^2}{8\pi}\sum_{i\neq j}^{\mathcal{N}}[\ln(|z_i-z_j|^2+|\epsilon|^2)\\&+\ln(|\Omega_i-\Omega_j|^2+|\epsilon|^2)]+\frac{\beta^2}{4\pi}\sum_{i,j}^{\mathcal{N}}\ln(|z_i-\Omega_j|^2+|\epsilon|^2)\\&-\frac{\mathcal{N}\beta^2}{4\pi}\ln(|\epsilon|^2)-\frac{\beta^2}{2}\sum_{i=1}^{\mathcal{N}}\int_{z_i,C_i}^{\Omega_i}\delta(\xi-\zeta)\,\mathrm{d}\zeta_\mu\bigg\}.\end{aligned} \tag{12}$$

The above expression corresponds to the partition function of a monopole gas in the grand canonical ensemble. Since the system is neutral, the μ-dependent terms cancel and expression (12) is (infrared) instability free. The first term in the exponent of expression (12) represents the interaction energy of the (imaginary) equally-charged gas 'monopoles'; the second, the interaction energy of the non-uniform background 'monopoles'; the third, the interaction energy of these two configurations. The fourth and fifth terms represent the corresponding self-energies and must be eliminated, respectively, by the fugacity renormalization [3, 5]

$$\alpha=\alpha_0(|\epsilon|^2)^{-\beta^2/4\pi} \tag{13}$$

and by the definition of the normal ordering factor as

$$\mathscr{F}(z)=\exp\left\{\frac{\beta^2}{2}\int_{z,C}^{\Omega}\mathrm{d}\xi_\mu\int_{z,C}^{\Omega}\delta(\xi-\zeta)\,\mathrm{d}\zeta_\mu\right\}. \tag{14}$$

A different assumption on the nature of the $\mathcal{N}$ curves C_i would just change the definition of the renormalization factor $\mathscr{F}(z)$, and the resulting vacuum functional remains path independent. It is dependent only on the 'dipole' localization.

We could require the independence on the choice of the string endpoint (Ω), and thus the absence of a limit-ordering prescription for the thermodynamical limit. To this end, we introduce an average over all possible Ω_i points in a finite volume $\mathscr{V}=\pi\mathscr{R}^2$. With this assumption, the potential is written as

$$\mathscr{U}(x)=\lim_{\mathscr{V}\to\infty}\mathscr{U}(x)_{\mathscr{V}} \tag{15}$$

where

$$\mathscr{U}(x)_{\mathscr{V}}=\mathrm{e}^{\beta\Sigma(x)_{\mathscr{V}}} \tag{16}$$

and

$$\Sigma(x)_{\mathcal{V}} = \frac{1}{\mathcal{V}} \int d^2\Omega\, \Sigma(x, \Omega). \tag{17}$$

In this way, the condition of independence on the choice of the Ω point introduces a uniform background of 'negative' charged 'monopoles'. The charge density is now given by

$$\tilde{\rho}(z; z_1; \ldots; z_{\mathcal{N}})_{\mathcal{M}} = \mathrm{i}\beta \mathcal{V}^{-1} \sum_{j=1}^{\mathcal{N}} \int d^2\Omega_j \int_{z_j, C_j}^{\Omega_j} \varepsilon_{\mu\nu} \partial^{\nu}_{(\xi)} \delta(\xi - z)\, d\xi^{\mu} \tag{18}$$

and we obtain for the vacuum functional ($\hat{z} = z/\mathcal{R}$)

$$\begin{aligned} \tilde{\mathscr{Z}}[0]_{\mathcal{M}} = \lim_{\mathcal{N}=0} \frac{(-\alpha_0)^{\mathcal{N}}}{\mathcal{N}!} \left(\frac{\mathcal{V}}{\pi}\right)^{\mathcal{N}} \int \prod_{i=1}^{\mathcal{N}} \mathcal{F}(\hat{z}_i)\, d^2\hat{z}_i \exp\bigg\{ \mathcal{N} \frac{\beta^2}{4\pi} \sum_{i=1}^{\mathcal{N}} |\hat{z}_i|^2 \\ - \frac{\beta^2}{8\pi} \sum_{i \neq j}^{\mathcal{N}} \ln[e^{3/2}(|\hat{z}_i - \hat{z}_j|^2 + |\hat{\epsilon}|^2)] - \mathcal{N} \frac{\beta^2}{8\pi} \ln|\hat{\epsilon}|^2 \\ - \frac{1}{2}\left(\frac{\beta}{\mathcal{V}}\right)^2 \sum_{i=1}^{\mathcal{N}} \int d^2\Omega\, d^2\Omega' \int_{z_i, C}^{\Omega} d\xi_\mu \int_{z_i, C}^{\Omega'} \delta(\xi - \zeta)\, d\zeta_\mu \bigg\}. \end{aligned} \tag{19}$$

Introducing the fugacity renormalization as [3]

$$\alpha = \alpha_0 (e^{3/2} |\hat{\epsilon}|^2)^{-\beta^2/8\pi} \tag{20}$$

and renormalizing the interaction Lagrangian density with

$$\mathcal{F}(z) = \exp\left\{ \frac{1}{2}\left(\frac{\beta^2}{\mathcal{V}}\right) \int d^2\Omega\, d^2\Omega' \int_{z,C}^{\Omega} d\xi_\mu \int_{z,C}^{\Omega'} \delta(\xi - \zeta)\, d\zeta_\mu \right\} \tag{21}$$

we obtain the following identity:

$$\tilde{\mathscr{Z}}[0]_{\mathcal{M}} \equiv \mathscr{Z}[0]_{\mathscr{C}} \tag{22}$$

where $\mathscr{Z}[0]_{\mathscr{C}}$ is the grand partition function of the charge-screened Liouville gas obtained in [3].

We can interpret the identity given by (22) as being the result of the symmetry under the interchange of charges and 'monopoles'; i.e. in the Euclidean space, two real equal point charges interact, via the Coulomb potential, in exactly the same way as two imaginary equally-charged 'monopoles' interact.

In order to show the existence of a translationally invariant ground state, we will consider the vacuum expectation value $\langle 0|\Box\phi(x)|0\rangle$ [3]. To obtain the equation of motion we use the standard variational principle on the action written in terms of $\Sigma(x)_{\mathcal{V}}$. Extending the quantum locality of exponential functions of $\Sigma(x)$ to the classical level imposing the condition $\delta\Sigma = \delta\phi$ [7], we obtain the following quantum equation of motion:

$$\Box\phi(x) = -\alpha\beta :e^{\beta\Sigma(x)_{\mathcal{V}}}: - \frac{1}{\mathcal{V}} \int d^2z :e^{\beta\Sigma(z)_{\mathcal{V}}}: . \tag{23}$$

In exactly the same way as in [3], we can easily show that

$$\lim_{\mathcal{V}\to\infty} \langle 0| :e^{\beta\Sigma(x)_{\mathcal{V}}}: |0\rangle_{\mathcal{M}} \equiv \lim_{\mathcal{V}\to\infty} \langle 0| :\exp\left\{ -\frac{\beta}{\mathcal{V}} \int d^2\Omega \int_x^{\Omega} \partial_\mu \Phi(z)\, dz^\mu \right\}: |0\rangle_{\mathscr{C}} = \text{constant} \tag{24}$$

where $\Phi(x)$ is the Liouville field. Using the equation of motion (23) we obtain, in the thermodynamical limit, the following 'free field weak condition'

$$\langle 0|\Box\phi(x)|0\rangle = 0 \tag{25}$$

which implies the existence of a translationally invariant ground state [2, 3].

The equivalence given by identity (24) is nothing but a consequence of the symmetry under the interchange of charges and 'monopoles'. We can obtain a general one-to-one mapping between arbitrary correlation functions of $\Sigma(x)$ and of the Liouville field $\Phi(x)$, making the following correspondences:

$$\left\{\begin{array}{ll}\Phi(x) & \Leftrightarrow \Sigma(x)_\phi = \int_{x,c}^{\infty} \varepsilon_{\mu\nu}\partial^\nu\phi(z)\,\mathrm{d}z^\mu \\ \Sigma(x)_\Phi = \int_{x,C}^{\infty} \varepsilon_{\mu,\nu}\partial^\nu\Phi(z)\,\mathrm{d}z^\mu \Leftrightarrow \phi(x) & \end{array}\right\}. \tag{26}$$

In this way, for any arbitary functional $\mathfrak{F}$ we obtain

$$\langle 0|\mathfrak{F}\{\phi, \Sigma_\phi\}|0\rangle_{\mathcal{M}} \equiv \langle 0|\mathfrak{F}\{\Sigma_\Phi, \Phi\}|0\rangle_{\mathcal{C}} \tag{27}$$

and the equivalence of the two theories is established.

The charge screening, induced by the uniform background, can be considered as the effect of an external potential interacting with the Liouville field through the coupling with a topological current

$$\mathcal{J}(x)_\mu = \frac{\beta}{\pi}\varepsilon_{\mu\nu}\partial^\nu\Phi(x) \tag{28}$$

whose associated conserved charge is given by

$$\mathcal{Q} = \lim_{\mathcal{R}\to\infty}\frac{\beta}{\pi}\{\Phi(\mathcal{R}) - \Phi(-\mathcal{R})\}. \tag{29}$$

Since the partition function (19) describes an infrared stable statistical system confined in a volume $\mathcal{V} = \pi\mathcal{R}^2$, we can use the following Green function

$$D(z)_{\mathcal{R}} = \lim_{\epsilon\to\infty}\left\{-\frac{1}{4\pi}\ln\left[\frac{|z|^2+|\epsilon|^2}{\mathcal{R}}\right]\right\} \tag{30}$$

which satisfies the Dirichlet boundary condition on $|z| = \mathcal{R}$. The vacuum functional (19) can be written as

$$\mathcal{Z} = \mathcal{Z}_0^{-1}\int \mathcal{D}\Phi\,\exp\left(-\int \mathrm{d}^2z\{\tfrac{1}{2}(\partial_\mu\Phi(z))^2 + \alpha_0\,\mathrm{e}^{\beta\Phi(z)}\,\mathrm{e}^{(\beta^2\tau/4\pi)|z|^2}\}\right) \tag{31}$$

where $\tau = \mathcal{N}^2/\mathcal{R}$ is regarded as a constant number density [8] in order to fulfil the neutrality condition. Rescaling the field variable,

$$\phi = \Phi + \frac{\beta\tau}{4\pi}|z|^2 \tag{32}$$

in such a way that it does not change the homotopy classes characterized by the topological charges $\mathcal{Q}$, we obtain from (31)

$$\mathcal{Z} = \mathcal{Z}_0^{-1}\int \mathcal{D}\phi\,\exp\left(-\int \mathrm{d}^2z\left\{\tfrac{1}{2}(\partial_\mu\phi(z))^2 - \frac{\pi}{\beta}\mathcal{A}(z)_\mu\mathcal{J}(z)^\mu + \alpha_0\,\mathrm{e}^{\beta\phi(z)}\right\}\right) \tag{33}$$

where $\mathscr{D}\phi = \mathscr{D}\Phi$, and the external potential $\mathscr{A}(x)_\mu$ is given by

$$\mathscr{A}(x)_\mu = \frac{\beta\tau}{2\pi}\,\varepsilon_{\mu\nu}x^\nu. \tag{34}$$

In the calculation of expression (33), a term proportional to $(\mathscr{A}_\mu)^2$ was disregarded.

We must note that, contrary to what occurs in the sine-Gordon-like systems [8, 9], the Liouville potential cannot be fermionized. In this case, the associated statistical system is composed of equal real point charges [3]. Consequently, no fermionic content emerges in the theory and the conserved current $\mathscr{J}(x)_\mu$ acquires a pure topological meaning. The charge screening in this translation-invariant system can be regarded as a two-dimensional exotic quantum effect [10].

Finally, we will make some remarks about the relation between the Liouville gas and the one-component plasma.

In the two-dimensional one-component plasma [8, 9], the resulting interparticle force can be viewed as the interaction between equal imaginary point charges. The corresponding Boltzmann factor is given by [7]

$$\Gamma_{\mathrm{ocp}}(z_1;\ldots;z_{\mathcal{N}}) = \exp\left\{\tfrac{3}{8}\gamma e^2\mathcal{N}^2 - \tfrac{1}{4}\mathcal{N}e^2\gamma\ln\left(\frac{\mathscr{R}^2}{L^2}\right) - \tfrac{1}{2}\gamma\mathcal{N}e^2\sum_{i=1}^{\mathcal{N}}\frac{|z_i|^2}{\mathscr{R}^2} + \tfrac{1}{2}\gamma e^2\sum_{i>j}^{\mathcal{N}}\ln\frac{|z_i-z_j|^2}{\mathscr{R}^2}\right\} \tag{35}$$

where $\gamma = (\mathscr{K}_{\mathrm{B}}\mathscr{T})^{-1}$, L is an arbitrary scale and e is the particles' charge. From (19), we see that the Boltzmann factor of the Liouville gas is given by

$$\Gamma(z_1;\ldots;z_{\mathcal{N}}) = \exp\left\{-\frac{3}{8}\left(\frac{\beta^2}{2\pi}\right)\mathcal{N}^2 + \mathcal{N}\frac{\beta^2}{8\pi}\ln\left(\frac{\mathscr{R}^2}{\epsilon^2}\right) + \mathcal{N}\frac{\beta^2}{4\pi}\sum_{i=1}^{\mathcal{N}}\frac{|z_i|^2}{\mathscr{R}^2} - \frac{\beta^2}{4\pi}\sum_{i>j}^{\mathcal{N}}\ln\frac{|z_i-z_j|^2}{\mathscr{R}^2}\right\} \tag{36}$$

and is related to the one-component plasma via

$$\Gamma(z_1;\ldots;z_{\mathcal{N}}) = \{\Gamma_{\mathrm{ocp}}(z_1;\ldots;z_{\mathcal{N}})\}^{-1} \tag{37}$$

provided we make the following identification: $\beta^2/2\pi = e^2/\mathscr{K}_{\mathrm{B}}\mathscr{T}$.

In [8, 9], the fermionized version of the one-component plasma leads to the mass operator $\pi = \Psi^*_{\{1\}}\Psi_{\{2\}}$ [7, 11], whose bosonized form is given by

$$\pi(x) = :\mathrm{e}^{\mathrm{i}\beta\Phi(x)}: \tag{38}$$

and the resulting fermionic Lagrangian is complex.

Using the symmetry under the interchange of charges and 'monopoles', together with (37), we conclude that the one-component plasma partition function can also be generated by the real (Euclidean) 'monopole' system. This theory is obtained from the Lagrangian density (1) making the change $\Sigma(x) \to \mathrm{i}\Sigma(x)$.

Taking this into account, the fermionized one-component plasma partition function can also be given by the superconducting gap operator $\sigma = \Psi_{\{1\}}\Psi_{\{2\}}$ [7, 11], whose bosonized expression is given by

$$\sigma(x) = :\mathrm{e}^{\mathrm{i}\beta\Sigma(x)}:. \tag{39}$$

The fact that, in the Euclidean space, the operator $\sigma(x)$ is real, must support some reasonable physical interpretation. As an example, in a future publication we intend to show that the connection between the one-component plasma and the probability distribution of Laughlin wavefunction [12] can be interpreted in terms of a well defined Euclidean fermionic theory.

Acknowledgments

The author is indebted to E C Marino and M Doria for clarifying conversations and suggestions. The hospitality at the Physics Institute of the Universidade Federal Fluminense is gratefully acknowledged. This work was supported by Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ).

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